Geometry: 7.4-7.5 Notes

NAME______________________

<u>7.4 Properties of Special Parallelograms</u>________________________________ <u>Date:</u>________________

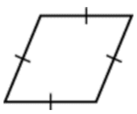
Define Vocabulary:

rhombus

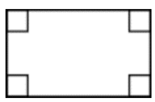
rectangle

square

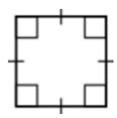
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.

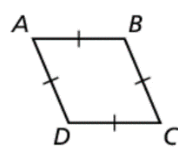


A **square** is a parallelogram with four congruent sides and four right angles.

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

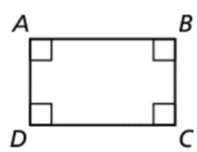
ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

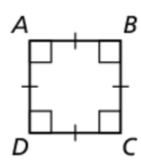
ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

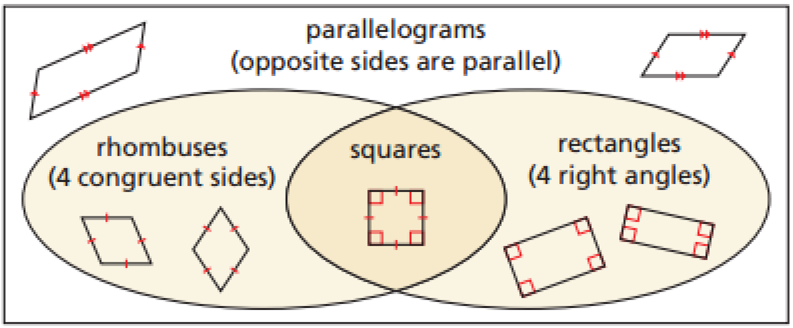


Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a squar e if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.





Examples: Use Properties of Special Quadrilaterals.

1. **WE DO**

For any rectangle *ABCD*, decide whether the statement is always or sometimes true. Explain your reasoning.

a. $AB = BC$

b. $AB = CD$

2. **YOU DO**

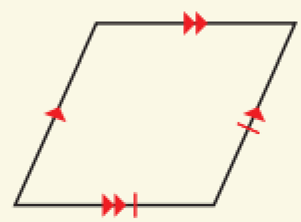
a. For any square JKLM, is it always or sometimes true that $\overline{JK} \perp \overline{KL}$? Explain your reasoning.

b. For any rectangle EFGH, is it always or sometimes true that $\overline{FG} \cong \overline{GH}$? Explain your reasoning.

Examples: Classifying special quadrilaterals.

3. **WE DO**

Classify the special quadrilateral. Explain your reasoning.



4. **YOU DO**

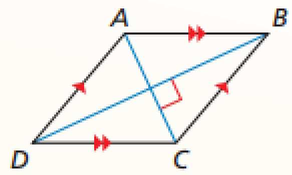
Classify a quadrilateral with four congruent sides and for congruent angles.

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

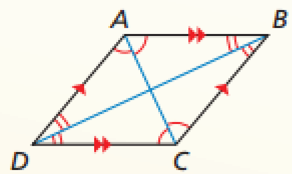


Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

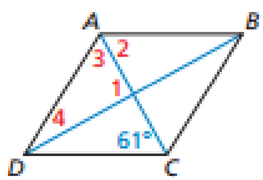
Proof Exs. 73 and 74, p. 395



Examples: Finding angle measures in a rhombus.

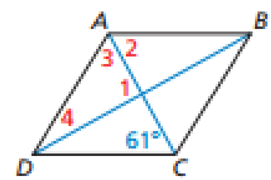
5. **WE DO**

Find the $m\angle ABC$ and $m\angle ACB$ in rhombus *ABCD*.

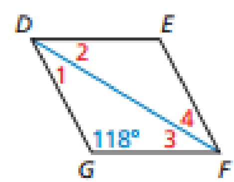


YOU DO

6. Find the $m\angle ADC$ and $m\angle BCD$ in rhombus *ABCD*.



7. Find the measures of the numbered angles in rhombus *DEFG*.

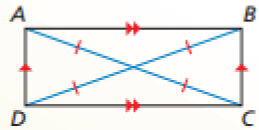


Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

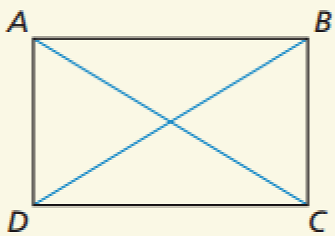
Proof Exs. 87 and 88, p. 396



Examples: Finding diagonal lengths in a rectangle.

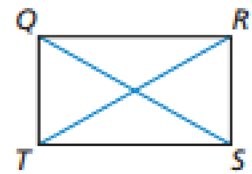
8. **WE DO**

In rectangle $ABCD$, $AC = 7x - 15$ and $BD = 2x + 25$. Find the lengths of the diagonals of $ABCD$.



9. **YOU DO**

QS = 4x – 15 and RT = 3x + 8, find the lengths of the diagonals in *QRST*.



Assignment	

7.5 Properties of trapezoids and kites____ Date:_____________

Define Vocabulary:

trapezoid

bases

base angles

legs

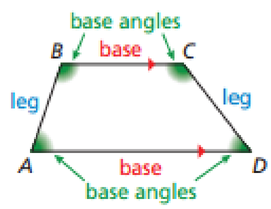
isosceles trapezoid

kite

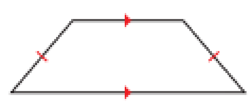
Using Properties of Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

Base angles of a trapezoid are two consecutive angles whose common side is a base. A trapezoid has two pairs of base angles. For example, in trapezoid $ABCD$, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.



If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



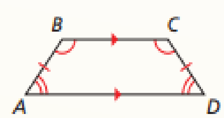
Isosceles trapezoid

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

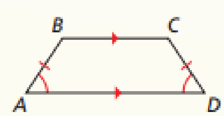


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof Ex. 40, p. 405

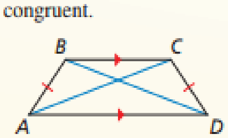


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

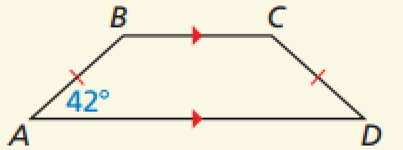
Proof Ex. 51, p. 406



Examples: Using properties of Isosceles Trapezoids.

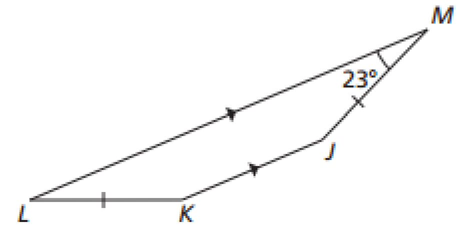
1. **<u>WE DO</u>**

ABCD is an isosceles trapezoid, and $m\angle A = 42°$. Find $m\angle B$, $m\angle C$, and $m\angle D$.



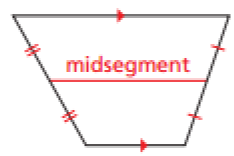
2. **<u>YOU DO</u>**

Find the measure of $\angle K\ and\ \angle L$.



Using the Trapezoid Midsegment Theorem

Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs. The theorem below is similar to the Triangle Midsegment Theorem (Thm. 6.8).



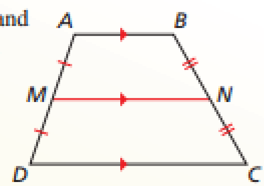
Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

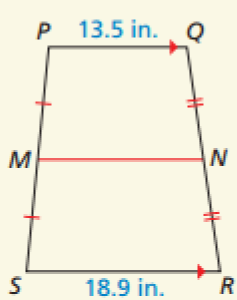
Proof Ex. 49, p. 406



Examples: Using the midsegment of the trapezoid

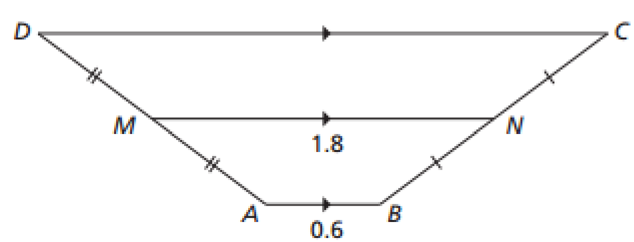
3. **<u>WE DO</u>**

In the diagram, $\overline{MN}$ is the midsegment of trapezoid *PQRS*. Find *MN*.



4. **<u>YOU DO</u>**

Find the length of CD.

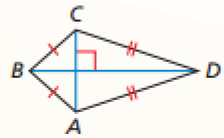


Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

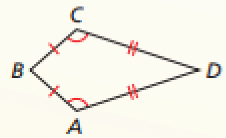


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

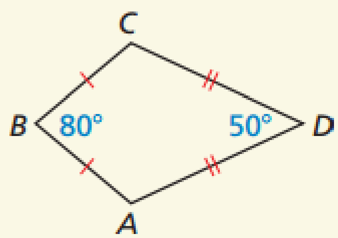
Proof Ex. 47, p. 406



Examples: Finding angle measures in a kite.

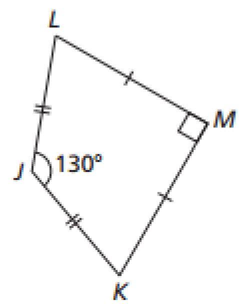
5. **<u>WE DO</u>**

Find $m\angle C$ in the kite shown.



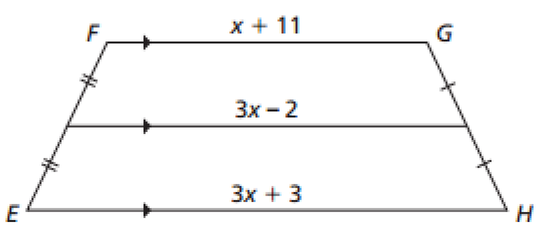
6. **<u>YOU DO</u>**

Find the measure of $\angle K$ and $\angle L$.

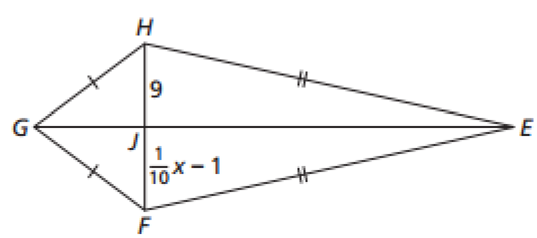


Examples: Use the properties to find the value of x.

7. **<u>WE DO</u>**



8. **<u>YOU DO</u>**



Assignment	